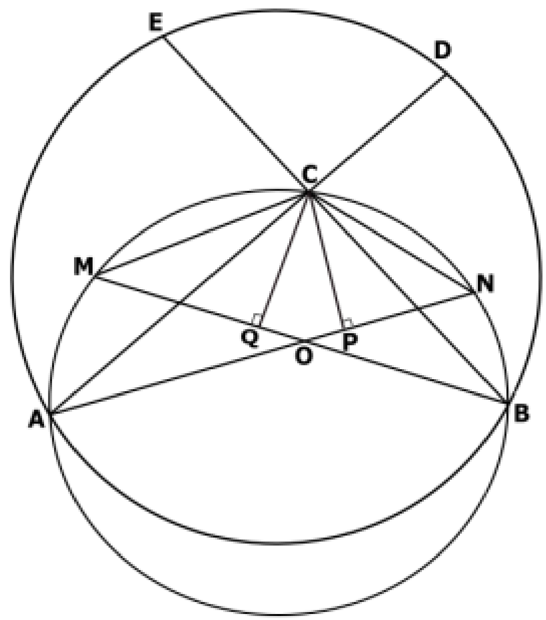


In the picture, two circles (one small and one big) intersect each other at A & B. C is any point on the major arc AB of the smaller circle. AC & BC are joined and produced to meet the bigger circle at D & E respectively. M & N are points on minor arcs AC & BC respectively of smaller circle such that CE = CM and CD = CN. AN & BM intersect at O. CP ⊥ AN and CQ ⊥ BM are drawn. Prove: OP = OQ.

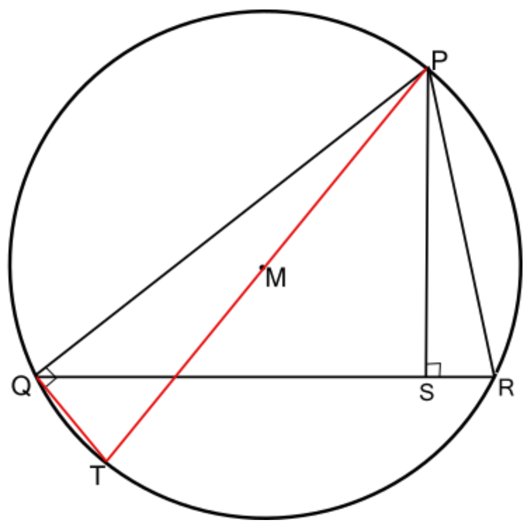
Question framed by
DR. M. RAJA CLIMAX
Founder Chairman,
CEOA Group of Institutions

Solution :

Before giving the proof, let us bring out a common feature about altitude as below.

In Δ PQR, if PS is an altitude to the side QR then,

$PS = \frac{PQ \times PR}{d}$ where 'd' is the diameter of the circumcircle. This is proved as follows.



Construction :

Draw the diameter PT through the circumcentre 'M'. Join QT.

Now, in $\Delta\ PSR$ & ΔPQT

$\angle PSR = \angle PQT = 90°$.

$\angle PRS = \angle PTQ$ (angles in the same segment)

$\therefore \Delta PSR$ & ΔPQT are similar.

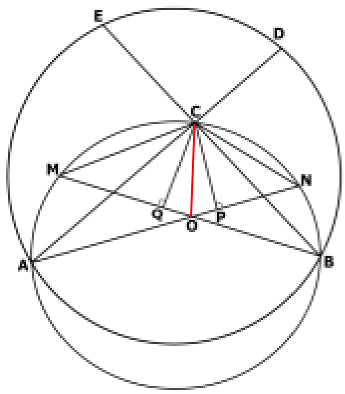
$$\frac{PS}{PQ} = \frac{SR}{QT} = \frac{PR}{PT}$$

$\mathrm{PR} \times PQ = \mathrm{PS} \times PT$

ie $\mathrm{PS} = \frac{PR \times PQ}{PT}$

ie Altitude $\mathrm{PS} = \frac{PR \times PQ}{d}$

Now, let us take the given problem



Construction :

Join OC

⇒

Applying the above formula

$CP = \frac{CM \times CN}{d}$ ----------------------(1)

And CQ $= \frac{CM \times CB}{d}$ ----------------(2)

But $\mathrm{CM} \times CB = CE \times CB$ ---------- (3)

And $\mathrm{CN} \times CA = CD \times CA$ --------- (4)

And $\mathrm{CA} \times CD = CE \times CB$ --------- (5)

∴ (1), (2), (3), (4) & (5) ⟶

CQ = CP

⇒ $\Delta\, COQ$ & ΔCOP are congruent [One side & Hypotenuse are equal]

∴ $OP = OQ$ ----------------- Proved.

****